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## ON COMMUTATIVE HYPER *BE*-ALGEBRAS

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**Abstract.** In this paper, we introduce commutative hyper *BE*-algebra and study it in detail. We show that every commutative (row diagonal, column row, very thin) hyper *BE*-algebra is a *BE*-algebra.

### 1. Introduction and Preliminaries

H. S. Kim and Y. H. Kim introduced the notion of a *BE*-algebra as a generalization of a dual *BCK*-algebra [4]. S. S. Ahn and et al. introduced the notions of terminal sections of a *BE*-algebras and gave some characterization of commutative *BE*-algebras in terms of lattices order relations and terminal sections [1]. A. Rezaei and et al. show that a commutative implicative *BE*-algebra is equivalent to the commutative self-distributive *BE*-algebra. Also, they proved that every Hilbert algebra is a self-distributive *BE*-algebra and commutative self-distributive *BE*-algebra is a Hilbert algebra and show that cannot remove the conditions commutativity and self-distributivity [7].

The hyper algebraic structure theory was introduced in 1934, by F. Marty at the 8th congress of Scandinavian Mathematicians [5]. Hyper-structures have many applications to several sectors of both pure and applied sciences. Y. B. Jun and et al. applied the hyper-structures to *BCK*-algebras and introduced the notion of a hyper *BCK*-algebra which is a generalization of *BCK*-algebra and investigated some related properties [3].

Recently, A. Radfar and et al. introduced the notion of hyper *BE*-algebra and defined some types of hyper-filters in hyper *BE*-algebras. They showed that under special condition hyper *BE*-algebras are equivalent to dual hyper *K*-algebras [6].

In this paper we characterize the relation between dual hyper *K*-algebras and commutative hyper *BE*-algebras and some types of commutative hyper *BE*-algebras and characterization of *RD/CR/V*-hypercommutative *BE*-algebras are state. We show that every commutative *RD*-hyper *BE*-algebra of order 3 is a commutative *BE*-algebra.

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**Definition 1.1.** [2] Let  $H$  be a nonempty set and  $\circ : H \times H \rightarrow P^*(H)$  be a hyper-operation. Then  $(H; \circ, 0)$  is called a hyper  $K$ -algebra, if it satisfies the following axioms:

- (HK<sub>1</sub>)  $(x \circ z) \circ (y \circ z) < x \circ y$ ,
- (HK<sub>2</sub>)  $(x \circ y) \circ z = (x \circ z) \circ y$ ,
- (HK<sub>3</sub>)  $x < x$ ,
- (HK<sub>4</sub>)  $x < y$  and  $y < x$  imply that  $x = y$ ,
- (HK<sub>5</sub>)  $0 < x$ , for all  $x, y, z \in H$ .

Where  $x < y$  is defined by  $0 \in x \circ y$ .

**Theorem 1.1.** [2] Let  $H$  be a hyper  $K$ -algebra. Then

- (i)  $x \in x \circ 0$ ,
- (ii)  $0 \in 0 \circ x$ , for all  $x \in H$ .

**Definition 1.2.** [6] Let  $H$  be a nonempty set and  $\circ : H \times H \rightarrow P^*(H)$  be a hyper-operation. Then  $(H; \circ, 1)$  is called a hyper  $BE$ -algebra, if it satisfies the following axioms:

- (HBE<sub>1</sub>)  $x < 1$  and  $x < x$ ,
- (HBE<sub>2</sub>)  $x \circ (y \circ z) = y \circ (x \circ z)$ ,
- (HBE<sub>3</sub>)  $x \in 1 \circ x$ ,
- (HBE<sub>4</sub>)  $1 < x$  implies  $x = 1$ , for all  $x, y, z \in H$ .

A hyper- $BE$ -algebra is said to be

- (i) row hyper  $BE$ -algebra (for short,  $R$ -hyper  $BE$ -algebra), if  $1 \circ x = \{x\}$ , for all  $x \in H$ ,
- (ii) column hyper  $BE$ -algebra (for short,  $C$ -hyper  $BE$ -algebra), if  $x \circ 1 = \{1\}$ , for all  $x \in H$ ,
- (iii) diagonal hyper  $BE$ -algebra (for short,  $D$ -hyper  $BE$ -algebra), if  $x \circ x = \{1\}$ , for all  $x \in H$ ,
- (iv) thin hyper  $BE$ -algebra (for short,  $T$ -hyper  $BE$ -algebra), if that is  $RC$ -hyper  $BE$ -algebra,
- (v) very thin hyper  $BE$ -algebra (for short,  $V$ -hyper  $BE$ -algebra), if that is  $RCD$ -hyper  $BE$ -algebra,

$(H; \circ, 1)$  is called a dual hyper K-algebra if satisfies  $(HBE_1)$ ,  $(HBE_2)$  and the following axioms:

$$(DHK_1) \quad x \circ y < (y \circ z) \circ (x \circ z),$$

$$(DHK_4) \quad x < y \text{ and } y < x \text{ imply that } x = y, \text{ for all } x, y, z \in H.$$

Where the relation " $<$ " is defined by  $x < y \Leftrightarrow 1 \in x \circ y$ . For any two nonempty subsets  $A$  and  $B$  of  $H$ , we define  $A < B$  if and only if there exist  $a \in A$  and  $b \in B$  such that  $a < b$  and

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b.$$

**Theorem 1.2.** [6] *Let  $H$  be a hyper BE-algebra. Then*

- (i)  $A \circ (B \circ C) = B \circ (A \circ C)$ ,
- (ii)  $A < A$ ,
- (iii)  $1 < A$  implies  $1 \in A$ ,
- (iv)  $x < y \circ x$ ,
- (v)  $x < y \circ z$  implies  $y < x \circ z$ ,
- (vi)  $x < (x \circ y) \circ y$ ,
- (vii)  $z \in x \circ y$  implies  $x < z \circ y$ ,
- (viii)  $y \in 1 \circ x$  implies  $y < x$ , for all  $x, y, z \in H$  and  $A, B, C \subseteq H$ .

**Corollary 1.1.** [6] *Every dual hyper K-algebra is a hyper BE-algebra.*

**Theorem 1.3.** [6] *Let  $H$  be a CD-hyper BE-algebra. Then*

- (i)  $x \circ (y \circ x) = \{1\}$ ,
- (ii)  $z \in x \circ y$  implies  $y \circ z = \{1\}$ ,

for all  $x, y, z \in H$ .

**Theorem 1.4.** [8] *Let  $X$  be a commutative BE-algebra. Then  $x \leq y$  and  $y \leq x$  implies  $x = y$ , for all  $x, y \in X$ .*

## 2. On commutative hyper BE-algebras

**Definition 2.1.** A hyper BE-algebra (dual hyper K-algebra)  $H$  is said to be commutative if  $(x \circ y) \circ y = (y \circ x) \circ x$ , for all  $x, y \in H$ .

**Example 2.1.** (i). Let  $H = \{1, a, b\}$ . Define the hyper-operations " $\circ_1$ " as follows:

$\circ_1$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1, b\}$	$\{1, a, b\}$	$\{1, a\}$
b	$\{1, a, b\}$	$\{a\}$	$\{1, a, b\}$

Then  $(H, \circ_1)$  is a commutative hyper BE-algebra.

(ii). Define the hyper operation " $\circ$ " on  $\mathbb{R}$  as follows:

$$x \circ y = \begin{cases} \{y\} & \text{if } x = 1 \\ \mathbb{R} & \text{otherwise} \end{cases}$$

Then  $(\mathbb{R}; \circ, 1)$  is a commutative hyper BE-algebra.

**Lemma 2.1.** Let  $H$  be a commutative hyper BE-algebra. Then  $H$  satisfies in  $(DHK_1)$ .

*Proof.* Let  $H$  be a commutative hyper BE-algebra and  $x, y, z \in H$ . Then

$$\begin{aligned} (x \circ y) \circ ((y \circ z) \circ (x \circ z)) &= (x \circ y) \circ (x \circ ((y \circ z) \circ z)) \\ &= (x \circ y) \circ (x \circ ((z \circ y) \circ y)) \\ &= (x \circ y) \circ ((z \circ y) \circ (x \circ y)) \\ &= (z \circ y) \circ ((x \circ y) \circ (x \circ y)). \end{aligned}$$

Now, using  $(HBE_1)$ , we have  $1 \in (x \circ y) \circ (x \circ y)$ .

Also, by  $(HBE_2)$ ,

$$1 \in (z \circ y) \circ ((x \circ y) \circ (x \circ y)).$$

Thus

$$1 \in (x \circ y) \circ ((y \circ z) \circ (x \circ z)).$$

Therefore  $(DHK_1)$  holds.  $\square$

**Theorem 2.1.**  $H$  is a commutative dual hyper K-algebra if and only if  $H$  is a commutative hyper BE-algebra and satisfies in  $(DHK_2)$ .

*Proof.* Let  $H$  be a commutative dual hyper K-algebra. Using Corollary 1.1,  $H$  is a commutative hyper BE-algebra.

Conversely, let  $H$  be a commutative hyper BE-algebra, satisfies in condition  $(DHK_2)$  and  $x, y, z \in H$ . Then by Lemma 2.1,  $(DHK_1)$  holds. Therefore  $H$  is a dual hyper K-algebra.  $\square$

The following example show that condition  $(DHK_2)$  in Theorem 2.1, is necessary.

**Example 2.2.** Let  $H = \{1, a, b\}$ . Define the hyper-operation " $\circ$ " as follows:

$\circ$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{1, a, b\}$
b	$\{1, a, b\}$	$\{1, b\}$	$\{1, a, b\}$

Then  $H$  is a commutative hyper BE-algebra. Since  $a < b$  and  $b < a$ ,  $H$  is not a hyper K-algebra.

**Proposition 2.1.** Let  $H$  be a commutative R-hyper BE-algebra. Then  $x \circ y = y \circ x = \{1\}$  implies  $x = y$ .

*Proof.* Let  $x \circ y = y \circ x = \{1\}$ . Since  $H$  is a commutative R-hyper BE-algebra,

$$\{x\} = 1 \circ x = (y \circ x) \circ x = (x \circ y) \circ y = 1 \circ y = \{y\}.$$

Therefore  $x = y$ .  $\square$

**Lemma 2.2.** Let  $H$  be a commutative C-hyper BE-algebra. Then

- (i)  $H$  is a commutative CD-hyper BE-algebra,
- (ii)  $y \in 1 \circ x$  implies  $1 \circ x \subseteq 1 \circ y$ ,
- (iii)  $y \in 1 \circ x$  if and only if  $x \in 1 \circ y$ ,

for all  $x, y \in H$ .

*Proof.* (i). Let  $x \in H$ . Then by commutativity,  $x \circ x \subseteq (1 \circ x) \circ x = (x \circ 1) \circ 1 = 1 \circ 1 = \{1\}$ . Thus  $H$  is a commutative D-hyper BE-algebra.

(ii). Let  $y \in 1 \circ x$ . Using Theorem 1.2 (viii),  $y < x$  and so  $1 \in y \circ x$ . Since  $y \in 1 \circ x$ , by Theorem 1.3(ii),  $x \circ y = \{1\}$  and by commutativity,

$$1 \circ x \subseteq (y \circ x) \circ x = (x \circ y) \circ y = 1 \circ y.$$

Hence  $1 \circ x \subseteq 1 \circ y$ .

(iii). Let  $y \in 1 \circ x$ . By (ii),  $1 \circ x \subseteq 1 \circ y$ . By  $(HBE_3)$ ,  $x \in 1 \circ x \subseteq 1 \circ y$ . Thus  $x \in 1 \circ y$ . Similarly,  $x \in 1 \circ y$  implies  $y \in 1 \circ x$ .  $\square$

**Corollary 2.1.**  $H$  is a commutative C-hyper BE-algebra if and only if  $H$  is a commutative CD-hyper BE-algebra.

**Lemma 2.3.** Let  $H$  be a commutative CR-hyper BE-algebra. Then

- (i)  $H$  is a commutative V-hyper BE-algebra,

- (ii) If  $x \circ y = \{1\}$ , then  $(y \circ x) \circ x = \{y\}$ ,
- (iii) If  $x \circ y = \{1\}$ , then  $y \circ x = \{z\}$  and  $y \circ z = \{y\}$ , for some  $z \in H$ ,

for all  $x, y \in H$ .

*Proof.* (i). By Corollary 2.1,  $H$  is a commutative CRD-hyper BE-algebra and so it is a commutative V-hyper BE-algebra.

(ii). If  $x \circ y = \{1\}$ , then  $(y \circ x) \circ x = (x \circ y) \circ y = \{1\} \circ y = \{y\}$ .

(iii). Let  $x \circ y = \{1\}$ . At first we prove that  $y \circ x$  is a singleton set. Let  $a, b \in y \circ x$ . By (ii),  $a \circ x \subseteq (y \circ x) \circ x = \{y\}$ . Thus  $a \circ x = \{y\}$ . By a similar way,  $b \circ x = \{y\}$ . By (i),  $H$  is a V-hyper BE-algebra and so it is a CD-hyper BE-algebra. Now, using Theorem 1.3 (ii) and since  $a \in y \circ x$ , we have  $x \circ a = x \circ b = \{1\}$ . By (ii),  $(a \circ x) \circ x = \{a\}$ . Thus

$$\{a, b\} \subseteq y \circ x = (a \circ x) \circ x = \{a\}.$$

Hence  $a = b$  and  $y \circ x$  is a singleton set. Now, let  $y \circ x = \{z\}$ . By using (ii),  $y \circ z = y \circ (y \circ x) = \{y\}$ .  $\square$

**Theorem 2.2.** Every commutative CR-hyper BE-algebra is a commutative BE-algebra.

*Proof.* Let  $H$  be a commutative CR-hyper BE-algebra. Let  $a, b \in x \circ y$ . By Lemma 2.3 (i),  $H$  is a V-hyper BE-algebra and so is a CD-hyper BE-algebra. By Theorem 1.3 (ii) and since  $a, b \in x \circ y$ , we can see that  $y \circ a = y \circ b = \{1\}$ . Now, using Lemma 2.3 (iii),

$$a \circ y = \{c\}, c \circ y = \{a\}, b \circ y = \{d\}, \text{ and } d \circ y = \{b\} \text{ for some } c, d \in H.$$

Since  $d = b \circ y \subseteq (x \circ y) \circ y = (y \circ x) \circ x$ , there is  $t_1 \in y \circ x$ , such that  $d \in t_1 \circ x$ . By Theorem 1.3 (ii), we imply that  $x \circ d = \{1\}$ . In a similar way,  $c = a \circ y \subseteq (x \circ y) \circ y = (y \circ x) \circ x$ , there is  $t_2 \in y \circ x$ , such that  $c \in t_2 \circ x$ . By Theorem 1.3 (ii), we get  $x \circ c = \{1\}$ . By  $(HBE_2)$ ,

$$b \circ a \subseteq b \circ (x \circ y) = x \circ (b \circ y) = x \circ d = \{1\}.$$

$$a \circ b \subseteq a \circ (x \circ y) = x \circ (a \circ y) = x \circ c = \{1\}.$$

Thus  $a \circ b = b \circ a = \{1\}$ . Now, using Proposition 2.1,  $a = b$ . Therefore  $x \circ y$  is a singleton set for every  $x, y \in H$  and so  $H$  is a commutative BE-algebra.  $\square$

### 3. Characterization of commutative hyper BE-algebra of order 3

From now on,  $H$  is a commutative hyper BE-algebra of order 3.

**Lemma 3.1.** Let  $H$  be a hyper BE-algebra. Then

- (i)  $1 \notin 1 \circ x$ , for all  $1 \neq x \in H$ ,

(ii)  $1 \in x \circ 1$ , for all  $x \in H$ .

*Proof.* (i). Let  $x \neq 1$  and  $x \in H$ .  $1 \in 1 \circ x$  implies  $1 < x$ . By  $(HBE_4)$ ,  $x = 1$ , which is a contradiction.

(ii). The proof is clear by using  $(HBE_1)$ .  $\square$

**Theorem 3.1.** *Let  $H$  be a commutative  $D$ -hyper BE-algebra of order 3. Then  $H$  is a commutative CD-hyper BE-algebra of order 3.*

*Proof.* Let  $H = \{1, a, b\}$  be a commutative  $D$ -hyper BE-algebra and it is not a C-hyper BE-algebra. Then  $a \circ 1 \neq \{1\}$  or  $b \circ 1 \neq \{1\}$ . Without loss of generality, let  $a \circ 1 \neq \{1\}$ . By  $(HBE_1)$ ,  $1 \in a \circ 1$ . Thus  $a \circ 1 = \{1, a\}$ ,  $\{1, b\}$  or  $\{1, a, b\}$  and so we have three cases:

Case 1: If  $a \circ 1 = \{1, a\}$ , then

$$1 \circ a \subseteq 1 \circ (a \circ 1) = a \circ (1 \circ 1) = a \circ 1 = \{1, a\}.$$

By Lemma 3.1,  $1 \notin 1 \circ a$  and so  $1 \circ a = \{a\}$ . Now, by commutativity,

$$\{1\} = a \circ a = (1 \circ a) \circ a = (a \circ 1) \circ 1 = \{1, a\} \circ 1 = \{1, a\},$$

which is a contradiction. Thus  $a \circ 1 \neq \{1, a\}$ .

Case 2: If  $a \circ 1 = \{1, b\}$ , then by Lemma 3.1(ii) and  $(HBE_2)$ ,

$$\{1, b\} = a \circ 1 \subseteq a \circ (b \circ 1) = b \circ (a \circ 1) = b \circ \{1, b\} = b \circ 1 \cup b \circ b = 1 \cup b \circ 1.$$

Hence  $b \circ 1 = \{1, b\}$ . Which is a contradiction.

Case 3: If  $a \circ 1 = \{1, a, b\}$ , then by commutativity,

$$(1 \circ a) \circ a = (a \circ 1) \circ 1 = \{1, a, b\} \circ 1 \supseteq a \circ 1 = \{1, a, b\}.$$

Now, by Lemma 3.1,  $1 \circ a = \{a\}$  or  $\{a, b\}$ . Since  $1 \circ a = \{a\}$ , we have

$$(1 \circ a) \circ a = a \circ a = \{1\}.$$

Thus  $1 \circ a \neq \{a\}$  and so  $1 \circ a = \{a, b\}$ . Also,

$$\{1, a, b\} = (1 \circ a) \circ a = \{a, b\} \circ a = a \circ a \cup b \circ a = 1 \cup b \circ a.$$

Thus  $\{a, b\} \subseteq b \circ a$ . Also, since  $1 \circ a = \{a, b\}$ , by Theorem 1.2 (viii),  $b < a$  and so  $1 \in b \circ a$ . Hence  $b \circ a = \{1, a, b\}$ . Also, by  $1 \circ a = \{a, b\}$  and Lemma 2.2(iii), we imply that  $a \in 1 \circ b$  and so  $1 \circ b = \{a, b\}$ . Now, since  $H$  is a  $D$ -hyper BE-algebra and by  $(HBE_2)$ ,

$$b \circ a \subseteq b \circ (1 \circ b) = 1 \circ (b \circ b) = 1 \circ 1 = \{1\}.$$

Thus  $b \circ a = \{1\}$ , which is a contradiction.  $\square$

**Corollary 3.1.** *Three concepts - commutative C-hyper BE-algebra, commutative  $D$ -hyper BE-algebra and commutative CD-hyper BE-algebra - coincide.*

**Corollary 3.2.** *Every commutative RD-hyper BE-algebra of order 3 is a commutative BE-algebra.*

**Theorem 3.2.** *There exist two commutative V-hyper BE-algebra of order 3 up to isomorphism.*

*Proof.* Let  $H = \{1, a, b\}$  be a commutative V-hyper BE-algebra. Then by Theorem 2.2,  $H$  is a BE-algebra and so,

$$1 \circ 1 = \{1\}, 1 \circ a = \{a\}, 1 \circ b = \{b\}$$

and

$$a \circ 1 = b \circ 1 = a \circ a = b \circ b = \{1\}.$$

By Theorem 1.4,  $a$  and  $b$  are comparable, hence  $a < b$  or  $b < a$ . Case  $a < b$  and case  $b < a$  are isomorphic. So, without loss of generality let  $a < b$ . Then  $a \circ b = \{1\}$ . By commutativity, we have,

$$(b \circ a) \circ a = (a \circ b) \circ b = 1 \circ b = \{b\}.$$

If  $b \circ a = \{1\}$ , then  $(b \circ a) \circ a = 1 \circ a = \{a\}$ , which is a contradiction.

If  $b \circ a = \{a\}$ , then  $(b \circ a) \circ a = a \circ a = \{1\}$ , which is a contradiction.

Since  $H$  is a BE-algebra,  $b \circ a = \{b\}$ . Therefore  $H$  is a commutative V-hyper BE-algebra.

$\circ$	1	$a$	$b$
1	$\{1\}$	$\{a\}$	$\{b\}$
$a$	$\{1\}$	$\{1\}$	$\{1\}$
$b$	$\{1\}$	$\{b\}$	$\{1\}$

Now, if  $a$  and  $b$  are comparable, then  $a \circ b = \{a\}$  or  $\{b\}$ . If  $a \circ b = \{a\}$ , then by Theorem 1.3(ii),  $b \circ a = \{1\}$  and so  $b < a$ , which is a contradiction. Hence  $a \circ b = \{b\}$ . By a similar way  $b \circ a = \{a\}$ . So,  $H$  is a commutative V-algebra.

$\circ$	1	$a$	$b$
1	$\{1\}$	$\{a\}$	$\{b\}$
$a$	$\{1\}$	$\{1\}$	$\{b\}$
$b$	$\{1\}$	$\{a\}$	$\{1\}$

□

**Theorem 3.3.** *There exist three commutative D-hyper BE-algebra of order 3 up to isomorphism.*



*Proof.* By Theorem 3.1,  $H$  is a CD-hyper BE-algebra and so,

$$1 \circ 1 = 1 \circ a = a \circ a = b \circ 1 = b \circ b = \{1\}.$$

By Lemma 3.1(i),  $1 \notin 1 \circ a$ . Also, by  $(HBE_3)$ ,  $a \in 1 \circ a$ . Thus  $1 \circ a = \{a\}$  or  $\{a, b\}$ . By a similar way,  $1 \circ b = \{b\}$  or  $\{a, b\}$ . If  $1 \circ a = \{a\}$ , then  $1 \circ b = \{b\}$  (Since  $1 \circ b = \{a, b\}$  we have  $b \in 1 \circ a$ , which is a contradiction). Thus  $H$  is a commutative V-hyper BE-algebra. By Theorem 3.2, two commutative V-hyper BE-algebra exist.

Now, if  $1 \circ a = \{a, b\}$ , then by Lemma 2.2(iii),  $a \in 1 \circ b$  and so  $1 \circ b = \{a, b\}$ . By Theorem 1.3 (ii),  $1 \circ a = \{a, b\}$  implies  $b \circ a = \{1\}$  and  $1 \circ b = \{a, b\}$  implies  $a \circ b = \{1\}$ . Therefore  $H$  is a commutative hyper BE-algebra in the following table.

$\circ$	1	$a$	$b$
1	$\{1\}$	$\{a, b\}$	$\{a, b\}$
$a$	$\{1\}$	$\{1\}$	$\{1\}$
$b$	$\{1\}$	$\{1\}$	$\{1\}$

□

**Theorem 3.4.** (i). *There exist 889 commutative hyper BE-algebras of order 3 up to isomorphism.*

(ii). *There exist 68 commutative R-hyper BE-algebras of order 3 up to isomorphism.*

#### 4. Conclusion and future work

Now, in the following table we summarize the results of this paper and show that the number of all kinds of commutative hyper BE-algebras of order 3. We note that by Corollary 3.1, the three concepts - commutative D-hyper BE-algebra, commutative C-hyper BE-algebra and commutative CD-hyper BE-algebra - coincide. Also, by Theorem 2.2, Corollaries 3.1 and 3.2, every commutative RD/CR/V-hyper BE-algebra is a BE-algebra.

Type	Condition	Number
commutative hyper BE-algebra	$a < b$ and $b \not< a$	325
commutative hyper BE-algebra	$a < b$ and $b < a$	495
commutative hyper BE-algebra	$a \not< b$ and $b \not< a$	69
commutative hyper BE-algebra		889
commutative R-hyper BE-algebra	$a < b$ and $b \not< a$	30
commutative R-hyper BE-algebra	$a < b$ and $b < a$	24
commutative R-hyper BE-algebra	$a \not< b$ and $b \not< a$	14
commutative R-hyper BE-algebra		68
commutative D-hyper BE-algebra	$a < b$ and $b \not< a$	1
commutative D-hyper BE-algebra	$a < b$ and $b < a$	1
commutative D-hyper BE-algebra	$a \not< b$ and $b \not< a$	1
commutative D-hyper BE-algebra		3
(commutative RD-hyer)commutative BE-algebra	$a < b$ and $b \not< a$	1
(commutative RD-hyer)commutative BE-algebra	$a < b$ and $b < a$	0
(commutative RD-hyer)commutative BE-algebra	$a \not< b$ and $b \not< a$	1
(commutative RD-hyer)commutative BE-algebra		2

In the future work we will try to get some results on another type of commutative hyper BE-algebras and state some properties on this structure and investigate some relationships between them.

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